ON THE POLYNOMIAL LIMIT CYCLES OF POLYNOMIAL DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we deal with ordinary differential equations of the form $dy/dx = P(x, y)$ where $P(x, y)$ is a real polynomial in the variables x and y, of degree n in the variable y. If $y = \varphi(x)$ is a solution of this equation defined for $x \in [0, 1]$ and satisfying that $\varphi(0) = \varphi(1)$, we say that it is a periodic orbit. A limit cycle is an isolated periodic orbit in the set of all periodic orbits. If $\varphi(x)$ is a polynomial, then $\varphi(x)$ is called a polynomial solution.

We study the maximum number and the multiplicity of the polynomial limit cycles of $dy/dx = P(x, y)$ in function of n. We prove that this differential equation has at most n polynomial limit cycles and that this bound is sharp. If $n = 1$ (linear equation), or $n = 2$ (Riccati equation), we prove that the differential equation $dy/dx = P(x, y)$ has at most n polynomial limit cycles counted with their multiplicities. For $n = 3$ (Abel equation) we show that at most three polynomial limit cycles can exist whereas the multiplicity of a polynomial limit cycle can be unbounded.

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