4th Symposium on Planar Vector Fields

Lleida, 5 – 9 September 2016

http://www.ssd.udl.cat/sympo4.html

Contents

1 General information

The symposium is the fourth edition of the "Symposium of Planar Vector Fields" which began at Lleida in November of 1996. This symposium is dedicated to Professor Javier Chavarriga, founder of the research group "*Seminari de Sistemes Dinmics de la Universitat de Lleida*" and who left us in November 2005.

The main interest of the Symposium is the qualitative theory of planar vector fields and related topics, as the bifurcation analysis, limit cycles, periodic limit sets, singularities, desingularization, algebraic invariant curves, integrability, center - focus problem, isochronous centers, period maps, finite cyclicity, foliations, etc. The aim of the symposium is two-fold: to survey recent progress made in this field and to explore new directions.

This symposium is organized by Isaac A. García, Jaume Giné, Maite Grau and Susanna Maza.

1.1 Contact information and addresses

All the conferences will be of one hour and will be given in English. The talks of the symposium will take place at:
Room 1.04
Escola Politècnica Superior, Universitat de Lleida,
Avinguda Jaume II, 69; 25001 Lleida. (In front of the river Segre)
Tel. (34) 973 70 2779 Geo: N 41 36' 29.58, E 0 37' 23.1

Accommodation:
Hotel AC (http://achotels.marriott.com/hotels/ac-hotel-lleida)
C\ Unió, 8; 25002 Lleida.
(In the other side of the river Segre and located at 300 meters from the Escola Politècnica Superior),
Tel. (34) 973 28 3910

Lunches will take place at "Menjador Universitari" (aka. "Cafeteria") in the building next to the Escola Politècnica Superior (Av. Jaume II, 71).

The e-mail address of the symposium is symposium@matematica.udl.cat and the web page is http://www.ssd.udl.cat/sympo4.html.

Taxis in Lleida
http://www.loteutaxi.com/ (34) 973 22 3300
http://taxilleida.com/ (34) 973 20 3050 or (34) 680 20 3050

EPS = Escola Politècnica Superior.

1.2 Internet connection

Wifi user at the university:

If you are an *eduroam* user, you have access to wifi using the credentials of your university. See http://www.udl.cat/en/wifi.html for more information on eduroam.

You can also choose the SSID *CONGRES*. In such a case, the credentials for validation are

User:	sympopvf
Password:	Sp-2016

User at PC Labs at the university:

The credentials are the same as for wifi usage.
There are several PC Labs open in the campus that you can use:

- Laboratori L4 (EPS 0.08), groud floor at the EPS.

- **Room P1.02** in the building "Edifici Polivalent" next to EPS (Avda. Jaume II, 71), where we have lunch.

Room 0.01 (ground floor at the EPS) will also be open for the use of all the participants of the meeting.

The conference room is Room 1.04 (first floor at the EPS).

2 List of participants

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3 Symposium program

This is the latest version of the program. We will post any change on the panel in front of Room 1.04 at EPS (the conference room).

Coffee breaks will be at “Cafeteria”, in the building next to EPS.

The poster session, scheduled on Wednesday afternoon, will be in the hall in front of “Cafeteria”.

SYMPOSIUM PROGRAM

Conference Room: 1.04 EPS

	MONDAY 5th	**TUESDAY 6th**	**WEDNESDAY 7th**	**THURSDAY 8th**	**FRIDAY 9th**
10:00 – 11:00	REGISTRATION at **11:00h** in “Cafeteria”	D. Peralta-Salas	V. Romanovski	C. Christopher	D.S. Shafer
COFFEE BREAK					
11:45 – 12:45	A. Gasull	E. Ponce	J. Torregrosa	C. Pantazi	P. de Maesschalck
BREAK					
13:00 – 14:00	A. Buica	X. Zhang	S. Walcher	M.J. Álvarez	
LUNCH					
15:30 – 16:30	J. Villadelprat	**Social activity**	**Poster session**	V. Mañosa	
BREAK					
16:45 – 17:45	A. Cima	**Social activity**	J. Suárez Pérez del Río	J.T. Lázaro	
BREAK					
18:00 – 19:00	J.L. Bravo	**Social activity**	M. Caubergh		
21:00				**Social dinner**	

3.1 Social activities

We have programmed a visit to *San Miguel* factory on Tuesday afternoon. The visit is about 16h, but we will meet before in front of the EPS to go all together by bus. The duration of this guided visit is around two hours. *San Miguel* is a national company of beer. We will know in detail the process of development of their beers in a factory that has created the most innovative products of the company. We will finish the visit taking a taste of some beers. Please tell Susanna Maza if you are going to attend this visit at the latest on Monday. The cost of this activity is 2.50 euros for the bus to the factory, to be paid to Susanna Maza on Monday.

We have also programmed a **social dinner** on Thursday evening, at 21h.
Restaurant DAVALL (Teatre de La Llotja)
Av. Tortosa, 4. 25005 Lleida.
Tel. 628 30 3988.

Dinner is in charge of each of the attendees and costs 35 euros (IVA included) per person, to be paid to the member of the organization Susanna Maza during the coffee breaks on Monday and Tuesday.
MENU:
STARTERS (TO SHARE EACH 4)
Warm salad of goat cheese with sweet apple
Iberian acorn-fed ham Sanchez Romero Carvajal
Bread with tomato and olive oil
Grilled vegetables from Lleida with “romesco”
Fried calamari with smoked salt
Warm carpaccio of octopus with potato and vegetables
MAIN DISHES
Grilled turbot with asparagus and Orio sauce
Or
Grilled sirloin steak with vegetables
DESSERT
Pie with apple from Lleida and cream
DRINKS
Water, Soda
White wine: Sisquella - Clos Pons
Red wine: Alges - Clos Pons
Coffees, Infusions

After the dinner, there is the possibility to enjoy a drink at the price of 6 euros in the bar DAMOON, at the terrace on the top of the same building of the restaurant.

For more information on cultural activities in Lleida, you can check http://www.turismedelleida.com/.

4 Invited talks: titles and abstracts

María Jesús Álvarez Universitat de les Illes Balears (Spain)

Existence and non-existence of limit cycles for generalized Abel equations.

In this talk we summarize the results obtained in the last years about the existence of limit cycles in generalized Abel equations. This works have been done in collaboration with professors J.L. Bravo, M. Fernández (UEX) and R. Prohens (UIB).

We are dealing with the generalized Abel equation

$$\dot{x} = A(t)x^m + B(t)x^n = \left(\sum_{l=1}^{k_1} a_l A_l(t)\right) x^m + \left(\sum_{l=k_1+1}^{k_1+k_2} a_l A_l(t)\right) x^n \tag{1}$$

where A_n, A_m are the following trigonometric functions:

$$A_l = a_l \sin^{i_l}(t) \cos^{j_l}(t).$$

Denote by $\mathcal{H}$ the Hilbert number of the family (1), *i.e.* the supremum (it could be infinite) over $(a_l) \in \mathbb{R}^{k_1+k_2}$ of the number of limit cycles of the family. The aim of this work is to characterize the families inside equation (1) whose Hilbert number is 0, the families whose Hilbert number is 1 and the families whose Hilbert number is greater or equal to 2. This main objective is not finished yet but the majority part of the work is completely solved.

Let us explain more carefully the main goal. The first step in order to solve the problem is the result by Lins-Neto that proved that the bound of the number of limit cycles can not deppend only on the degrees n, m. Consequently, this bound must also deppend on the degrees of the trigonometric polynomials $A(t), B(t)$. In order to do that, we introduce a notation, that has proved to be very useful:

$$\mathcal{E} = \{\sin^i(t)\cos^j(t) \colon i, j \text{ even}\}, \qquad \mathcal{S} = \{\sin^i(t)\cos^j(t) \colon i \text{ odd}, j \text{ even}\},$$
$$\mathcal{C} = \{\sin^i(t)\cos^j(t) \colon i \text{ even}, j \text{ odd}\}, \quad \mathcal{O} = \{\sin^i(t)\cos^j(t) \colon i, j \text{ odd}\}.$$

Observe that $x = 0$ is always a periodic orbit of equation (1). Thus, the problem os characterizing the families with $\mathcal{H} = 0$ is equivalent to solve the center problem, *i.e.* the families in which every bounded solution is periodic. This problem is completely solved and the result is the following:

Theorem [Álvarez-Bravo-Fernández]: $\mathcal{H} = 0$ if and only if $A_l \in \mathcal{S} \cup \mathcal{O}$ for all l or $A_l \in \mathcal{C} \cup \mathcal{O}$ for all l.

Next step is characterizing the existence of limit cycles: $\mathcal{H} = 1$ means that, for any a_k the origin of equation (1) is the only limit cycle, except for some of them, for which it is a center. And $\mathcal{H} \geq 2$ means that there exist some values of a_k such that there exists a non-trivial limit cycle.

The problem of knowing which families have $\mathcal{H} = 1$ (and consequently $\mathcal{H} \geq 2$) is completely solved in the case that the function $A(t)$ (or $B(t)$) has only one trigonometric monomial. In order to solve the remaining cases, the following reduction has been proved:

Proposition [Álvarez-Bravo-Fernández]: In order to characterize the existence or non-existence of limit cycles of equation (1) it is enough with studying the equation

$$\dot{x} = (a_1A_1(t) + a_2A_2(t))x^m + (a_3A_3(t) + a_4A_4(t))x^n. \tag{2}$$

Inside the previous family most of the cases have been studied and characterized inside the class $\mathcal{H} = 1$ or $\mathcal{H} \geq 2$. In this talk we will devote special atention to the last case that has been proved (the $\mathcal{SCSC}$ one) and to the techniques used to proved it.

José Luis Bravo Trinidad Universidad de Extremadura (Spain)

TANGENTIAL AND ITERATED ZERO-DIMENSIONAL CENTERS.

Abstract. Given a polynomial $f \in \mathbb{C}[z]$ of degree m, let $z_1(t), \ldots, z_m(t)$ denote all algebraic functions defined by $f(z_k(t)) = t$. Given integers $n_1, \ldots, n_m$, the tangential center problem on zero-cycles asks to find all polynomials $g \in \mathbb{C}[z]$ such that $n_1 g(z_1(t)) + \ldots + n_m g(z_m(t)) \equiv 0$.

The persistent version of previous center problem asks to find all polynomials $g \in \mathbb{C}[z]$ such that $n_1 g(z_1(t, \epsilon)) + \ldots + n_m g(z_m(t, \epsilon)) \equiv 0$, for all $t, \epsilon \in \mathbb{C}$, where $z_1(t, \epsilon), \ldots, z_m(t, \epsilon)$ denote all algebraic functions defined by $f(z_k(t, \epsilon)) + \epsilon g(z_k(t, \epsilon)) = t$.

We will present some (partial) answers to both problems and their relation with the classical Center-Focus Problem.

Adriana Buică Babeş-Bolyai University of Cluj-Napoca (Romania)

On the time of first return of a perturbed periodic orbit

We consider a family of planar analytic vector fields which depend on a small parameter ε and such that the unperturbed system obtained for $\varepsilon = 0$ has a continua of periodic orbits in a region denoted $\mathcal{P}$. For a transversal section Σ we denote by $T^\Sigma(q, \varepsilon)$ the first return time to Σ of the orbit starting at $q \in \Sigma$. Consider now the following expansion

$$T^\Sigma(q, \varepsilon) = T_0(q) + \varepsilon T_1^\Sigma(q) + \varepsilon^2 T_2^\Sigma(q) + \ldots, \quad q \in \Sigma\,.$$

Assume also that there exists $m \geq 1$ such that the first m Melnikov functions vanish and that for some transversal section S, the functions T_0, T_1^S, ..., T_{m-1}^S are constant on S.

We succeeded to prove that, in this situation, these functions are the same constant for any transversal section. This assures that the critical points of $T^\Sigma(q, \varepsilon)$ are essentially the critical points of T_m^Σ.

Moreover, we proved that there exists an analytic first integral in $\mathcal{P}$ of the unperturbed system, $T_m : \mathcal{P} \to \mathbb{R}$ such that $T_m^\Sigma(q) = T_m(q)$ for any $q \in Sigma$ and any Σ. This assures that the critical points of T_m^Σ do not depend on the transversal section Σ.

Hence our results open the possibility to study the critical points of the first time return map in systems which are perturbations of a system with a period annulus.

This is a joint work with Jaume Giné and Maite Grau.

Magdalena Caubergh Universitat Autònoma de Barcelona (Spain)

Separatrix bifurcations in some 3-parameter families of planar cubic vector fields.

In this talk we report on joint works with J. Torregrosa and J. Llibre and new work by the author. Attention goes to the analytic study of separatrix bifurcations present in cubic differential equations

$$\begin{aligned}\dot{x} &= -y + ax^2 + bxy + cy^2 - y(x^2 + y^2),\\ \dot{y} &= x + ex^2 + fxy + gy^2 + x(x^2 + y^2),\end{aligned}$$

that have simultaneously a center in the origin and at infinity for $a, b, c, e, f, g \in \mathbb{R}$. In particular this study allows to complete analytically the topological classification of global phase portraits for this family of cubic differential equations.

Colin Christopher Plymouth University (United Kingdom)

ALGEBRAIC SOLUTIONS OF LOTKA VOLTERRA SYSTEMS: A GEOMETRICAL APPROACH.

Abstract: We revisit the classification of Moulin-Ollagnier of all Lotka-Volterra systems with algebraic solutions. Our aim is to understand better the underlying geometry of these solutions, with a view to giving a more "geometric" proof of the classification. Although we have not been able to complete the latter, we are able to give a number of interesting properties which give necessary conditions for all invariant curves of low degree. In many of these cases, we can also prove the sufficiency of these conditions geometrically. As an application, we discuss the extension of the monodromy technique to integrable critical points of Lotka-Volterra systems which do not lie at the intersection of the axes.

This is joint work with Wuria Hussein and Zhaoxia Wang.

Anna Cima Universitat Autònoma de Barcelona (Spain)

On the number of polynomial solutions of some Abel equations.

We investigate the number of polynomial solutions of the equation $q(t)\,\dot{x} = p_3(t)\,x^3 + p_2(t)\,x^2 + p_1(t)\,x + p_0(t)$ where $q(t)$ and $p_i(t)$ are polynomials in complex coefficients for $i = 0, 1, 2, 3$. When $p_2(t) = 0$ we prove that the above equation has at most seven polynomial solutions and that this bound is sharp. Also when the equation has a concrete solution $\bar{x}(t)$ and $k\bar{x}(t)$ with $k \notin \{1, -1\}$ we prove that at most we can get five polynomial solutions and that this bound also is sharp. In the proof we use some previous results on the polynomial solutions of $Q_1^p + Q_2^p + Q_3^p + Q_4^p = 0$ for some natural number p, where Q_i , $i = 1, 2, 3, 4$ are polynomials in one variable and also some tools of algebraic geometry.

This is a joint work with A. Gasull and F. Mañosas.

Peter De Maesschalck Hasselt University (Belgium)

Slow-fast Bogdanov-Takens bifurcations in an application.

Abstract:
We study a more degenerate version of the well-known slow-fast Van der Pol system, this time with a singularity that singularly unfolds as a Bogdanov-Takens bifurcation. We base ourselves on the local study performed in [1], complement it with a global geometric singular perturbation analysis (in [2]) to give a thorough view of all involved bifurcations such as Hopf, Homoclinic, SNIC bifurcations.

[1] De Maesschalck, P. and Dumortier, F., Slow-fast Bogdanov-Takens bifurcations, Journal of Differential Equations 250 (2) (2011), 10001025.

[2] De Maesschalck, P. and Wechselberger, M., Neural excitability and singular bifurcations, Journal of Mathematical Neuroscience 5 (16) (2015), 132.

Armengol Gasull Universitat Autònoma de Barcelona (Spain)

On the periods of the periodic solutions of non-autonomous periodic ODE.

Smooth non-autonomous T-periodic differential equations $x'(t) = f(t, x(t))$ defined in $\mathbb{R} \times \mathbb{K}^n$, where $\mathbb{K}$ is $\mathbb{R}$ or $\mathbb{C}$ and $n \geq 2$ can have periodic solutions with any arbitrary period S. We show that this is not the case when $n = 1$. We prove that in the real $\mathcal{C}^1$-setting the period of a non-constant periodic solution of the scalar differential equation is a divisor of the period of the equation, that is $T/S \in \mathbb{N}$. Moreover, we characterize the structure of the set of the periods of all the periodic solutions of a given equation. We also prove similar results in the one-dimensional holomorphic setting. In this situation the period of any non-constant periodic solution is commensurable with the period of the equation, that is $T/S \in \mathbb{K}$.

This talk is based on a joint work with A. Cima and F. Mañosas that is going to appear in *Differential and Integral Equations*.

J. Tomás Lázaro Universitat Politècnica de Catalunya (Spain)

Mixed dynamics in planar reversible diffeomorphisms.

Existence of open regions of structural instability was a crucial discovery for Bifurcation Theory. In this sense, the pioneering works of Newhouse [1970, 1979] were essential. In these works he proved that there exist open regions in the space of dynamical systems where systems with homoclinic tangencies are dense (in the $\mathcal{C}^r$-topology, $r \geq 2$). In particular, such domains exist in any neighbourhood around any planar diffeomorphism having an homoclinic tangency. This tangency, typically quadratic, can be governed by a one-parameter unfolding, and constitutes a codimension-1 bifurcation. Commonly, Newhouse regions refer not only to the open domains of dynamical systems but also to the domains in the corresponding parameter space. From the 70's there have been important contributions to develop and extend the work of Newhouse: Shilnikov, Gavrilov, Palis, Viana, Duarte, Tedeschini-Lalli, Yorke, Gonchenko, Turaev, Romero, and many others.

One of the main difficulties studying such regions in the space of dynamical systems is its extreme richness. They can exhibit coexistence of homoclinic tangencies of any (arbitrarily high) order, degenerate periodic orbits, etc. This means that to provide a complete description of their dynamics is almost an impossible target. Papers from Gavrilov, Shilnikov, Tedeschini-Lalli, Yorke, Gonchenko, Turaev among others, suggested as one of the most important properties in these Newhouse regions the coexistence of infinitely many periodic orbits (of the single or multiple round map) of different types of stability (attracting, repelling, saddle and elliptic). This leads to the concept of the so-called *Mixed dynamics*.

Its study in the framework of reversible maps was started (as far as we know) by a paper of Lamb and Stenkin in 2004. One important reason for such choice is that reversible maps can present both conservative and dissipative-like behaviours, making them very attractive. The final goal would be to prove what is called the *Reversible mixed dynamics conjecture*, which claims that planar reversible maps with mixed dynamics are generic in Newhouse regions where maps with symmetric homoclinic and /or heteroclinic tangencies are dense.

This talk is based on two papers [2012 Nonlinearity and 2016 - to be submitted] with people from Shilnikov group at the Nizhny Novgorod Lobachevsky University, namely, S. Gonchenko, V. Gonchenko, O. Stenkin, M. Gonchenko and A. Delshams (UPC).

Víctor Mañosa Universitat Politècnica de Catalunya (Spain)

Lie Symmetries and non-integrability of maps: the Cohen map case.

We consider the problem of characterizing the local $\mathcal{C}^m$-non-integrability, for some $m \in \mathbb{N}$, near elliptic fixed points of smooth planar measure preserving maps. Our criterion relates this non-integrability with the existence of some Lie Symmetries, which are some vector fields associated to the maps, together with the study of the finiteness of its periodic points. One of the steps in the proof uses the regularity of the period function on the whole period annulus for non-degenerate centers of planar vector fields, a question that is revisited. The obtained criterion can be applied to prove the local non-integrability of the Cohen map.

This is a joint work with Anna Cima and Armengol Gasull (Universitat Autònoma de Barcelona).

Chara Pantazi Universitat Politècnica de Catalunya (Spain)

On the non-integrability of some families of planar vector fields.

Abstract

We study the integrability/non integrability of planar polynomial vector fields using techniques of differential Galois Theory. We apply Ziglin-Morales-Ramis approach in order to decide the non-integrability of some families of foliations with rational coefficients coming from planar vector fields. Kovacic's algorithm will be used in some particular cases. We also present an approach which relate the order of the poles of the variational equations with the integrability of the foliation associated to a planar vector field.

This is a work in progress and in collaboration with P. Acosta-Humánez, T. Lázaro and J.J. Morales-Ruiz

Daniel Peralta-Salas ICMAT (Spain)

REALIZATION PROBLEMS FOR LIMIT CYCLES OF PLANAR POLYNOMIAL VECTOR FIELDS.

We show that for any finite configuration of closed curves $\Gamma \subset \mathbb{R}^2$, one can construct an explicit planar polynomial vector field that realizes Γ, up to homeomorphism, as the set of its limit cycles with prescribed periods, multiplicities and stabilities. The only obstruction given on this data is the obvious compatibility relation between the stabilities and the parity of the multiplicities. The constructed vector fields are Darboux integrable and admit a polynomial inverse integrating factor. Some open problems on the realization problem will be also discussed. This is based on joint work with Juan Margalef-Bentabol (J. Differential Equations 260 (2016) 3844–3859).

Jesús S. Pérez del Río Universidad de Oviedo (Spain)

(in collaboration with B. García, A. Lombardero and J. Llibre)

ON THE CLASSIFICATION OF QUASI-HOMOGENEOUS DIFFERENTIAL SYSTEMS IN DIMENSIONS TWO AND THREE

Quasi-homogeneous differential systems are very important in the study of dynamical systems. In fact, motion equations of many relevant problems of dynamics are of the quasi-homogeneous form, for example, Euler-Poisson equations, Kirchhoff equations, and so forth. These systems have significant properties (for example, all of them are integrable) and they have been studied from many different points of view (integrability, centers, normal forms, limit cycles).

But so far there was not an algorithm to build all quasi-homogeneous polynomial differential systems of a given degree until our work [1], that gives such algorithm and that we recall here. Using this algorithm we obtained the classification of the quasi-homogeneous planar systems of degree 2 and 3 and moreover we gave all analytic first integrals of these systems. Later, using our algorithm, other authors have obtained the systems of degrees 4 ([3]) and 5 ([4],[5]) and have also determined their phase portraits or their centers.

As a natural continuation of [1], in a recent work ([2]) we study the problem of the classification in a general way and we obtain the exact number of different forms of quasi-homogeneous but nonhomogeneous planar differential systems of an arbitrary degree n, proving a nice relation between this number and Euler's totient function, widely used in number theory.

Currently, we have started the study of three dimensional quasi-homogeneous systems in order to generalize our results about the planar case. We present here some new results about these systems.

References

[1] B. García, J. Llibre and J.S. Pérez del Río, *Planar quasi-homogeneous polynomial differential systems and their integrability*, J. Differ. Equ. **255**, 3185-3204 (2013).

[2] B. García, A. Lombardero and J.S. Pérez del Río, *Classification and counting of planar quasi-homogeneous differential systems through their weight vectors*, preprint (2015).

[3] H. Liang, J. Huang and Y. Zhao, *Classification of global phase portraits of planar quartic quasi-homogeneous polynomial differential systems*, Nonlinear Dyn. **78**, 1659-1681 (2014).

[4] Y. Tang, L. Wang and X. Zhang, *Center of planar quintic quasi homogeneous polynomial differential systems*, Discrete Contin. Dyn. Syst. **35**, 2177-2191 (2015).
[5] Xiong, Yanqin; Han, Maoan; Wang, Yong *Center problems and limit cycle bifurcations in a class of quasi-homogeneous systems*, Internat. J. Bifur. Chaos Appl. Sci. Engrg. 25 (2015), no. 10, 1550135, 11 pp.

Enrique Ponce University of Sevilla (Spain)

THE TEIXEIRA SINGULARITY DEGENERACY AND ITS BIFURCATION IN PWL SYSTEMS.

Abstract

Three-dimensional piecewise linear Filippov differential systems with a separation plane, having a two-fold point with invisible tangencies, sometimes referred as Teixeira's singularity, will be addressed. For some parameter values, the two vector fields involved at the two-fold are anti-parallel and then this singularity undergoes a compound bifurcation: there appears a sliding bifurcation involving a pseudo-equilibrium point and, simultaneously, a bifurcation associated to the birth of a crossing limit cycle. This singularity, first considered by Filippov, and its associated bifurcation have been recently studied in a series of papers, but a complete characterization is still lacking.

After determining a generic canonical form for the vector fields, we show how to characterize such a compound bifurcation in the case of piecewise linear systems. Several examples, including realistic applications, will be addressed.

The presentation comes from a joint work with Rony Cristiano and Daniel J. Pagano (both from UFSC Florianpolis, Brasil), and Emilio Freire (U. Sevilla).

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Valery Romanovski University of Maribor (Slovenia)

INVARIANTS AND TIME-REVERSIBILITY IN POLYNOMIAL SYSTEMS OF ODE'S

We present some results related to the theory of invariants of ordinary differential equations. Invariants of a group of orthogonal transformations of two-dimensional systems are considered in details. An algorithm to compute a generalizing set of invariants is given and an interconnection of the invariants and time-reversibility is shown. Some generalizations to the case of three-dimensional systems are discussed as well.

Douglas Shafer University of North Carolina at Charlotte (United States)

Dynamics of the time-delayed Duffing equation on the slow manifold.

Under certain generic conditions the asymptotic behavior of a delayed Duffing equation is governed by a system of ordinary differential equations on a two-dimensional manifold called the slow manifold. Possible behavior, including creation of limit cycles by the introduction of the delay, is explored. This work is an extension and partial correction of work presented in its early stages at Luminy several years ago.

Joan Torregrosa Universitat Autònoma de Barcelona (Spain)

NEW LOWER BOUNDS FOR $M(n)$ AND $H(n)$ FOR SMALL VALUES OF n.

A particular version of the 16th Hilbert's problem is to estimate the number, $M(n)$, of limit cycles bifurcating from a singularity of center-focus type. The problem to finding lower bounds for $M(n)$ for some concrete n can be done studying the cyclicity for different weak-foci or centers. Since a weak-focus with high order is the most current way to produce high cyclicity, we present systems with few monomials with the highest known weak-focus order. Christopher in [1] proved that under some assumptions the linear or quadratic parts of the Lyapunov constants with respect to the parameters give the cyclicity of an elementary center. In fact, it is proved that there exists, in the parameter space, a curve of weak-foci or order N that unfold N limit cycles. We will show a new approach, namely parallelization, to compute the linear parts of the Lyapunov constants. More concretely, it is showed that parallelization computes these linear parts in a shorter quantity of time than other traditional mechanisms, see [2, 3]. In [4] some simple families of even low degree with a weak-focus of order $n^2 + n - 2$ are presented. But it is difficult to prove that, perturbing with polynomials of degree n, this number of limit cycles can bifurcate from the weak-focus. For small degree n polynomial vector fields, we will present some improvements on the known lower bounds for $M(n)$. This approach can be also used to get new lower bounds for the Hilbert number, $H(n)$, studying the simultaneous bifurcation of centers with different period annuli. In this last case we only study the bifurcation from the singular points.

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Jordi Villadelprat Universitat Rovira i Virgili (Spain)

Unfoldings of saddle-nodes and their Dulac time.

Abstract

By unfolding a saddle-node, saddles and nodes appear. I will explain three different results. The first result (Theorem A) gives a uniform asymptotic expansion of the trajectories arriving at the node. Uniformity is with respect to all parameters, including the unfolding parameter bringing the node to a saddle-node and a parameter belonging to a space of functions. This is used for proving a regularity result (Theorem B) on the Dulac time (time of Dulac map) of an unfolding of a saddle-node. This result is a building block in the study of bifurcations of critical periods in a neighbourhood of a polycycle. Finally, I will explain how we apply Theorems A and B to the study of critical periods of the Loud family of quadratic centers and to prove that no bifurcation occurs for certain values of the parameters (Theorem C).

This is a joint work with P. Mardesic (Université de Bourgogne, France), D. Marín (UAB) and M. Saavedra (Universidad de Concepción, Chile)

Sebastian Walcher RWTH Aachen (Germany)

CHARACTERIZING PLANAR POLYNOMIAL VECTOR FIELDS WITH AN ELEMENTARY FIRST INTEGRAL.

Abstract. A well known theorem by Prelle and Singer implies that every polynomial planar vector field which admits a first integral that is elementary over the rational function field $\mathbb{C}(x, y)$ already admits a first integral of a rather special type, and a particular Darboux integrating factor. We show moreover that, apart from certain exceptional cases, such a vector field even admits a Darboux first integral and can be written in explicit form. This result uses earlier work by the authors and by Chavarrige et al. The exceptional cases are related to elements of norm one, resp. trace zero, in a certain algebraic extension of $\mathbb{C}(x, y)$, and a discussion indicates that they occur quite rarely. (This is joint work with Jaume Llibre and Chara Pantazi)

Xiang Zhang Shanghai Jiao Tong University (P.R.China)

PIECEWISE SMOOTH DIFFERENTIAL SYSTEMS WITH A CENTER: NORMAL FORM AND LIMIT CYCLE BIFURCATION.

In this talk I report our results on normal forms of piecewise smooth differential systems around a center called a Σ–center. Then we use this normal form to study limit cycle bifurcation from a Σ–center under small perturbation.

On the normal form we provide not only a topological normal form of a planar piecewise C^r smooth vector field with a Σ–center, but also the piecewise C^r smoothness of the homeomorphism. We also present examples showing the existence and nonexistence of the conjugacy of a planar piecewise Σ–center with its normal form.

5 Posters: titles and abstracts

Murilo Rodolfo Cándido Universitat Autònoma de Barcelona (Spain)

Averaging Theory For Finding Limit Cycles: A New Approach and Application.

The usual averaging theory reduces the computation of some periodic solutions of a system of ordinary differential equations, to find the simple zeros of an associated averaged function. When one of these zeros is not simple, i.e., the Jacobian of the averaged function in it is zero, the classical averaging theory does not provide information about the periodic solution associated to it. In this work we provide sufficient conditions in order that the averaging theory can be applied also to non-simple zeros for studying their associated periodic solutions. Additionally, we Apply this result for studying the zero–Hopf bifurcation in the Lorenz system.

This is a joint work with Jaume Llibre, Universitat Autònoma de Barcelona.

Alex Carlucci Rezende Universitat Autònoma de Barcelona (Spain)

CONFIGURATIONS OF INVARIANT HYPERBOLAS AND LINES IN QUADRATIC VECTOR FIELDS IN THE PLANE

ABSTRACT. Let **QSH** be the whole class of non-degenerate planar quadratic differential systems possessing at least one invariant hyperbola. We classify this family of systems, modulo the action of the group of real affine transformations and time rescaling, according to their geometric properties encoded in the configurations of invariant hyperbolas and invariant straight lines which these systems possess. The classification is given both in terms of algebraic geometric invariants and also in terms of affine invariant polynomials and it yields a total of 205 distinct such configurations. We have 162 configurations for the subclass $\mathbf{QSH}_{(\eta>\mathbf{0})}$ of systems which possess three distinct real singularities at infinity, and 43 configurations for the subclass $\mathbf{QSH}_{(\eta=\mathbf{0})}$ of systems which possess either exactly two distinct real singularities at infinity or the line at infinity filled up with singularities. The algebraic classification, based on the invariant polynomials, is also an algorithm which makes it possible to verify for any given real quadratic differential system if it has invariant hyperbolas or not and to specify its configuration of invariant hyperbolas and straight lines.

Antoni Ferragut Universitat Jaume I (Spain)

QUANTITATIVE ANALYSIS OF COMPETITION MODELS.

We study the planar Lotka-Volterra differential system, of competition between two species, having a saddle in the first quadrant. We show that, under certain conditions on the parameters of the system, one of the separatrices of the saddle divides the first quadrant into two, and depending on the initial conditions one of the species will extinct because the ω-limits are attracting nodes on the axes. We study the probability of the species of surviving depending on the initial choice of the parameters, providing an index κ.

José Ginés Espín Buendía Universidad de Murcia (Spain)

Minimal flows on nonorientable surfaces
(This is a work in collaboration with D. Peralta-Salas and G. Soler).

We investigate what surfaces admit minimal flows (i. e. flows with all its orbits dense). The case of orientable surfaces was solve, in 1998, by J. C. Benière who, in his Ph.D. Thesis, proved that all orientable surfaces which are not embeddable in the real euclidean plane posses a minimal C^∞ flow. However, up to our knowledge, the minimality of nonorientable surfaces has not been characterized so far.

In a recent work, we have fulfilled this gap for the case of nonorientable surfaces of finite genus and have advanced in the study of the infinite genus ones. This poster is intended to show the main results and ideas in that work.

Luiz F. S. Gouveia Universitat Autònoma de Barcelona (Spain)

Lower bound for the local cyclicity of quintic planar polynomial vector fields.

Hilbert early last century presented a list of problems that almost all of them are solved. One problem that remains open is the second part of the 16th Hilbert's problem: It is to determine the maximal number (named $H(n)$) of limit cycles, and their relative positions, of a planar polynomial systems of degree n

$$\begin{cases} \dot{x} &= -y + P_n(x,y), \\ \dot{y} &= \quad x + Q_n(x,y). \end{cases}$$

We are interested here in the local version of this Hilbert's 16th problem that consist in to provide the number $M(n)$ of small amplitude limit cycles bifurcating from an elementary center or an elementary focus. Clearly $M(n) \leq H(n)$. See more details in [Zol1995].

For $n = 2$, Bautin proved in [Bau] that $M(n) = 3$. Sibirskii in [Sib] proved that for cubic systems without quadratic terms there are no more than five limit cycles bifurcating from one critical point. In [Zol1995, Zol2015] Zoladek found an example where eleven limit cycles could be bifurcated from a single critical point of a cubic system and Christopher, with the technique presented in [Chr], gave a simpler proof of Zoladek's result perturbing a Darboux cubic center.

We prove that $M(5) \geq 32$ and we have numerical evidences that $M(5) \geq 33$. More concretely, we present a center such that we prove analytically that 32 limit cycles bifurcate from the origin, and another one such that, we can show, numerically, that 33 limit cycles bifurcate from the origin. We remark that this lower bound coincide with the value, $M(n) = n^2 + 3n - 7$, conjectured by Giné in [Gin2007]. The computations have been done using a generalization of the parallelization procedure, introduced by Liang and Torregrosa in [LiaTor], for finding the higher order terms in the perturbation parameters.

This is a joint work Joan Torregrosa (UAB).

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Lucile Mégret Université Pierre et Marie Curie/ INRIA Paris (France)

STUDIES OF THE PETROV MODULE FOR A GENERALIZED LIÉNARD INTEGRABLE SYSTEM

We use the Lambert function to develop a study for a family of integrable Liénard equations which displays a center. Every equations of this family admit an Hamiltonian H. First we prove a local Morse lemma inside the period annulus surrounding the center. Then we deduce an explicit operator of Gelfand-Leray associated to the Hamiltonian. This part is a generalisation of some results proved by Françoise and Xiao. Finally via the use of the Lambert function, we study the monotonicity of the Abelian integral for a generating family of the Petrov module, associated to the Hamiltonian.

Leonardo P. C. da Cruz Universitat Autònoma de Barcelona (Spain)

The Theorem of Bendixon-Dulac for piecewise differential systems

The classical Bendixson-Dulac Theorem provides a criteria to get upper bounds for the number of limit cycles in analytic differential systems. We extend this result to some classes of piecewise differential systems. In particular, we use this new result to the piecewise Liénard system

$$Z^{\pm} = \begin{cases} \dot{x} = y \\ \dot{y} = -x + \lambda^{\pm}(1 - x^2)y; \end{cases} \quad \text{if } (x, y) \in \Sigma^{\pm},$$

with $\Sigma^{\pm} = \{(x, y) : \pm x > 0\}$. We prove that this equation has exactly one limit cycle when $\lambda^+\lambda^- \geq 0$ and $(\lambda^+)^2 + (\lambda^-)^2 \neq 0$. Furthermore, we present the bifurcation diagram and it phase portrait when $\lambda^{\pm} \in \mathbb{R}$.

This is a joint work with Joan Torregrosa (Universitat Autònoma de Barcelona).

Salomón Rebollo-Perdomo Universidad del Bío-Bío (Chile)

Invariant algebraic curves for Kukles systems.

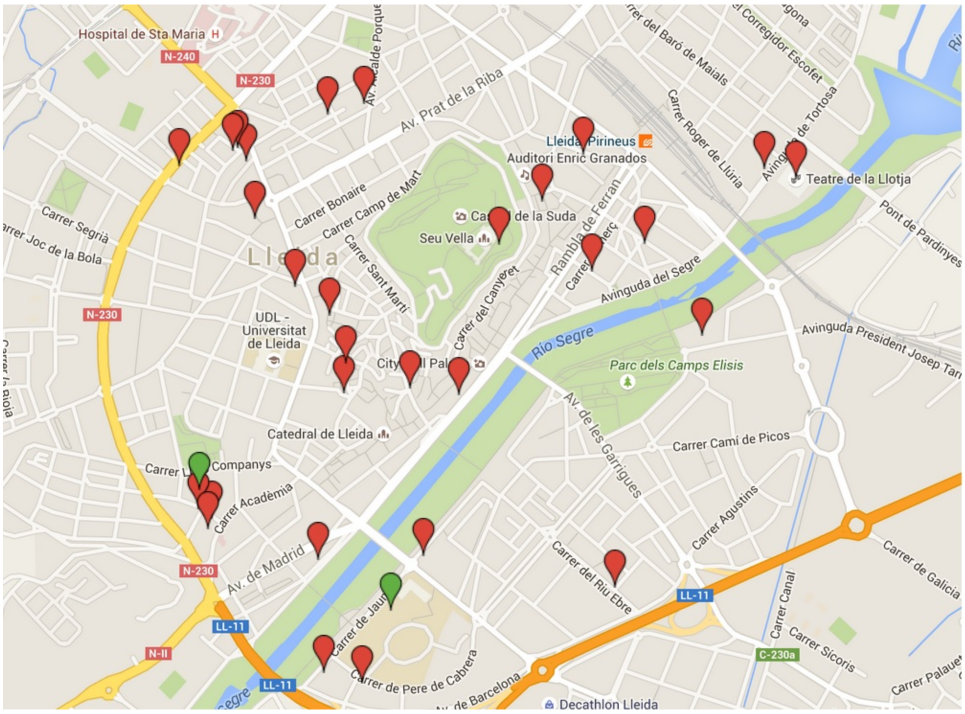
We give a upper bound for the degree of a class of transversal to infinity invariant algebraic curves for polynomial Kukles systems of arbitrary degree. We also study the roll that transversal to infinity invariant algebraic curves plays in the integrability of these systems.

Qinlong Wang Hezhou University (China)

HOPF CYCLICITY AND INTEGRABILITY FOR A 3D LOTKA-VOLTERRA SYSTEM

Abstract: The main objective of this paper is not only to find necessary and sufficient conditions for a three dimensional Lotka-Volterra system to have a center at the positive equilibrium, but also to determine the maximum number of limit cycles that can bifurcate from the fine focus restricted to the center manifold. Firstly the singular point quantities are computed and simplified to obtain necessary conditions for integrability, and Darboux method is applied to show the sufficiency. Then the Hopf bifurcation on the center manifold is investigated, from this, five and at most five small limit cycles from the equilibrium are obtained. To the best of our knowledge, this is the first example with five limit cycles for the cyclicity of 3D Lotka-Volterra systems.

6 List of restaurants



For some suggestions, check Map of Restaurants

- 1203
  Ronda Vella, 1
- 747 Gastrobar
  Alcalde Porqueres, 11
- Aggio
  Templers, 3
- Braseria Ca l'Isidre
  Alcalde Areny, 4
- Bokoto
  Magí Morera, 57
- Can Miquel
  Templers, 9
- Casa Martí
  Magdalena, 37

- Celler del Roser
  Cavallers, 24
- Celleret del Segre
  General Brito, 10
- Classual
  Humbert Torres, 7
- Click Menú
  Acadèmia, 40
- Creperie Bretonne
  Plaça Cervantes
- Davall (Teatre de La Llotja)
  Av. Tortosa, 4
- El Cau de Sant Llorenç
  Pl. Sant Josep, 4
- El Galeó
  Anselm Clavé, 17
- El Mirador dels Camps Elisis
  Av. President Josep Terradellas, 45-47
- Gastronomik 2.0
  Riu Ebre, 39
- Gourmet Real (Hotel Real)
  Av. Blondel, 22
- Iruña
  Sant Llorenç, 3
- LAntiquari, tapes de mercat
  Passeig de Ronda, 92
- La Capital
  Av. Estudi General, 1
- La Huerta
  Av. Tortosa, 7

- La Risueña
  Pere Cabrera, 12
- La Tagliatella
  Rovire Roure
- Macao
  Camp de Mart, 27
- Paddock
  Jaume II, 81
- Piñana
  Alcalde Fuster, 5
- Sangiovesa
  Balmes, 7
- Teresa Carles
  Ricard Vinyes, 5
- Vaporetto
  Magí Morera, 59